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Correlation functions in the Nagel–Schreckenberg model

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Abstract. We investigate the effect of the jamming transition on short-range correlation functions in the Nagel–Schreckenberg cellular automaton model of single-lane traffic. At high densities the structure of the time-dependent correlation functions is double peaked. One peak corresponds to moving cars, the other to blocked cars. The emergence of the latter peak as well as the occurrence of short-range anticorrelations behind the propagating peak is related to the jamming transition. At even higher densities the peak corresponding to moving cars disappears which is an indication of a superjamming transition.

1. Introduction

Particle hopping models in traffic have attracted considerable interest both in the physics and the traffic engineering community [1–3]. For one-dimensional single-lane traffic, the lane consists of L cells of equal size which can be either empty or occupied by a vehicle with velocity $v = 0, 1, \ldots, v_{\max}$. Motion takes place by hopping between cells. Particle hopping models can be divided into two classes according to their boundary conditions: periodic with conservation of the number of particles or open with injection and extinction of particles. In our investigations we consider periodic boundary conditions. Another distinguishing criterion is the manner in which sites are updated. The asymmetric stochastic exclusion process (ASEP), which was first solved by Derrida *et al* [4] for open boundary conditions and maximum velocity $v_{\max} = 1$, uses random sequential update. In the cellular automaton model developed by Nagel and Schreckenberg [5], however, all vehicles are handled in parallel during one timestep according to the following rules:

$$\text{increase } v \text{ by 1 if } v < v_{\max} \tag{1}$$

$$\text{decrease } v \text{ to avoid crashes with front cars} \tag{2}$$

$$\text{decrease } v \text{ by 1 with probability } p \text{ if } v > 0 \tag{3}$$

$$\text{move forward } v \text{ sites.} \tag{4}$$

Our considerations are based on the Nagel–Schreckenberg (NS) model with $v_{\max} = 5$, $p = 0.5$, as it shows a better congruence with real traffic data than the ASEP model. Due to the NS model–as it can also be observed in real traffic—the dynamics of the system can be resolved into three regimes depending on the car density ρ.

$0 < \rho \leqslant \rho_1$: free flow. Interactions between cars are rare, there is

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a very small probability that 'minijams' occur which immediately resolve.

$\rho_1 < \rho \leqslant \rho_2$: jamming. Typical for this stage is the coexistence of free flow and jamming [6]. In the spacetime diagram [5] we see that single jams spontaneously occur and dissolve after a while, but that there are also cars moving with maximum velocity $v_{\max}$.

$\rho_2 < \rho \leqslant 1$: superjamming. The whole system is congested, the jamming waves become connected and form an infinite wave [7].

This results not only from the original NS model [6, 7] but also from its cruise control limit [8], from its continuous limit [9], and from the model developed by Takayasu and Takayasu [10] which corresponds to the cruise control version of its deterministic limit for $v_{\max} = 1$.

It is obvious that the above characterization of free flow, jamming and superjamming is rather vague. In what way, for example, does a minijam in the free flow phase differ from a jam occurring in the jamming phase? In order to give a precise definition of free flow and jamming, Nagel and Paczuski [8] suggested considering a car to be jammed if it does not go with maximum velocity. In a site-oriented definition [7] a site is said to be in a jam, if two or more cars are within a window of five cells centred on a site.

These definitions are, nevertheless, arbitrary. What is still missing is a physical quantity which exactly determines free flow, jamming and superjamming. Accordingly, investigations were concerned in recent years with the transition from freely moving to jammed traffic, and with the question of whether an order parameter for this transition does exist.

In this context correlation functions which have already been used in the ASEP model [11–14] are of special interest. In the NS model as well, they are not only useful to get better analytical results [15–17]. As is known from statistical physics correlation functions are a powerful tool to investigate phase transitions and, therefore, they can give a better insight into the problem mentioned above. By investigating the spatial correlation function near the deterministic limit [18] and the correlation function in car-space [19] it turned out that the transition from freely moving to jammed traffic is not sharp but rather like a crossover.

So, the transition from jamming to superjamming is still an open question. This problem has already been handled for $p = 0.25$ by investigating the average number of jamming waves surviving at least until a time t [7]. Correlation functions, however, have not extensively been used in this context before.

In the following we consider the effect of the transition from freely flowing to jammed and from jammed to superjammed traffic, respectively, on short-range correlation functions depending on space and time. We furthermore deal with the question of whether free flow, jamming and superjamming can be detected with the help of short-range correlation functions.

2. Correlation function

The steady state correlation function on which our consideration is based has the form

$$C(i,t) = <\eta(i',t')\eta(i+i',t+t')> i',t' - \rho^2 \qquad (5)$$

where

$$\rho = \frac{\text{number of cars}}{\text{system size}}$$

is the car density and

$$\eta(i', t') = 1 \qquad \text{if site } i' \text{ is occupied at time } t'$$
$$\eta(i', t') = 0 \qquad \text{otherwise.}$$

Furthermore $< \cdots > i', t'$ describes the spatial and temporal average over all L sites i' and over times t' taken from our simulation of the steady state. In addition, ensemble averages were also carried out.

In order to have $C(i, t) = 0$ for the completely uncorrelated system we had to subtract the term ρ^2 in (5) as usual.

Normally we had to wait $10L$ timesteps (updates of the whole system), until the steady state was reached and then measured the correlation function. At the measurements themselves we took the average over 500 systems and 10 000 timesteps.

3. Results

It is important to mention that $C(i, t = 0)$ represents a kind of 'snapshot' of the lane whereas $C(i, t > 0)$ contains dynamical information. Therefore, we have to distinguish between correlation functions for $t = 0$ and those for $t > 0$ in the following.

For $t = 0$ the correlation function is symmetric showing a strong peak at $i = 0$ (figure 1(*a*)). For $t = 1, 2, 3$ a peak moves to the right for increasing time difference representing the motion of the cars (figures 1(*b*)–(*d*)). It can be seen that randomization has an influence on the peak's shape. The randomization probability is $p = 0.5$. Therefore, on average, half of the cars moving with $v_{\max}$ slow down at each timestep resulting in a broadening of the peak. The lower the density the more this broadening (which goes from the lattice site $i = t(v_{\max} - 1)$ to $i = tv_{\max}$) corresponds to what can be calculated from simple combinatorics. This can be explained by considering a certain site i. The probability that a car is slowed down according to rule (3) is $p = 0.5$. Going one timestep further, the probability that a car on i decreases its velocity due to (3) once again is 0.25, at the third timestep it is 0.125 and so on. This observation is confirmed at even larger time differences $t = 4$, 5, 6 in figure 2. At higher densities the asymmetry of the peak becomes more and more apparent as cars have to slow down in order to avoid crashes with the front cars.

At even higher densities ($\rho \approx 0.3$) the propagating peak disappears as shown in figure 3. The correlation becomes monotonously falling for all i with $t(v_{\max} - 1) \leqslant i \leqslant tv_{\max}$ and only a shoulder of the former peak can be seen. At densities $\rho \approx 0.4$ there is not even a trace of the propagating peak. Remarkably, these densities roughly coincide with the percolation density $\rho_p = 0.42$, Csányi and Kertész described (on the basis of their definition of jamming) the second transition density as where the isolated jamming waves start to percolate [7].

The disappearance of the propagating peak and the percolation of the isolated jamming waves are the motivation for the introduction of the expression 'superjamming' in order to describe this transition.

With increasing density a peak around $i = 0$ emerges and it develops a maximum at $i = -1$ corresponding to the hindrance the back car feels in the jam. In other words this peak describes the correlation with the back car. Accordingly, this peak does not disappear for very low densities, too. It just becomes nearly invisible and moves to the left for

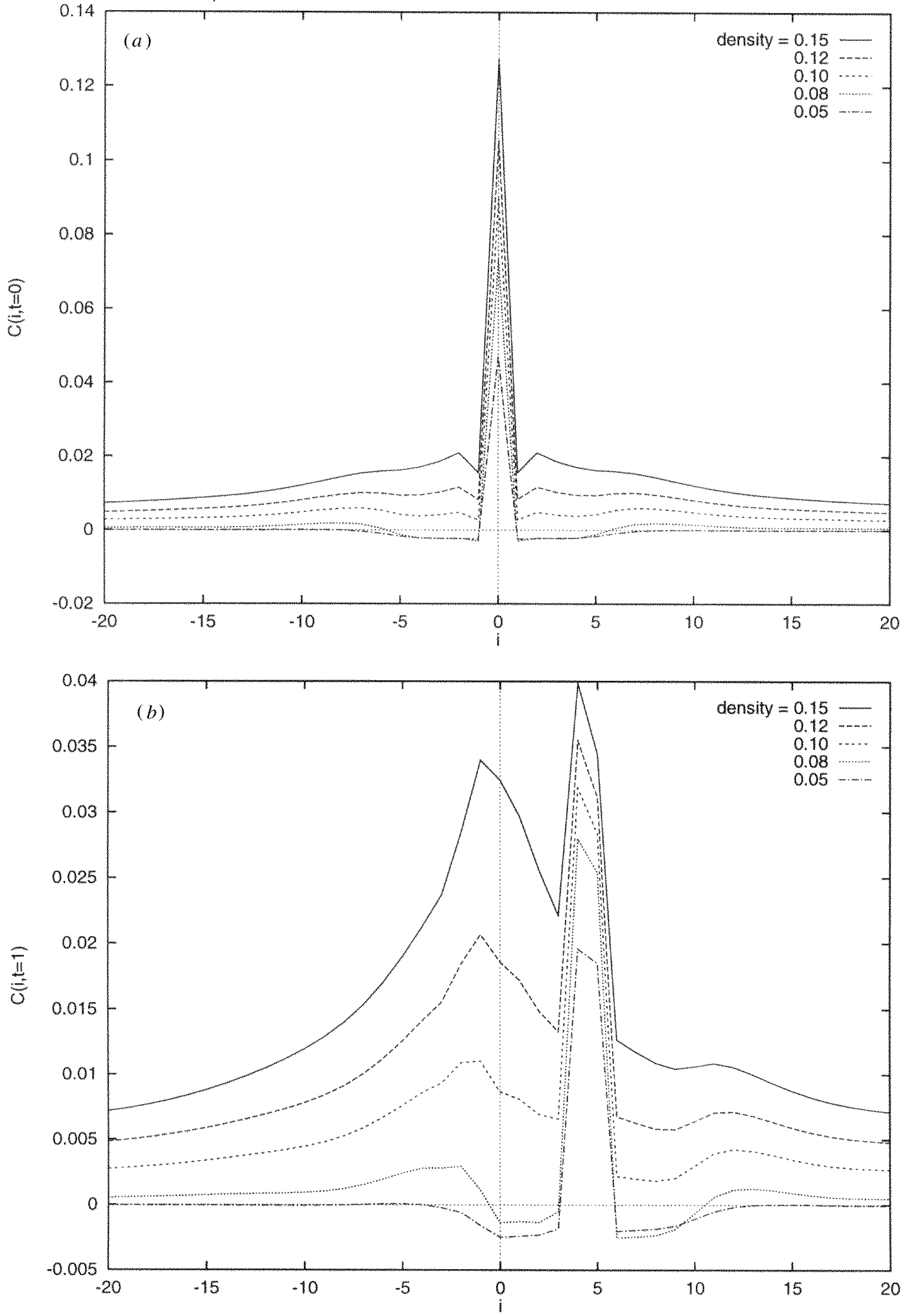


Figure 1. Correlation functions for the correlation times $t = 0, 1, 2, 3$ with the car densities $\rho = 0.05, 0.08, 0.1, 0.12, 0.15$ each ($L = 4096$). Whereas for $t = 0$ the correlation functions are symmetric showing a strong autocorrelation peak, for $t = 1, 2, 3$ the coexistence of the freely flowing and the jammed state can be seen. All diagrams have in common that coming from low densities anticorrelations develop in the neighbourhood of the autocorrelation peak. From a certain density on, however, they decrease and finally vanish.

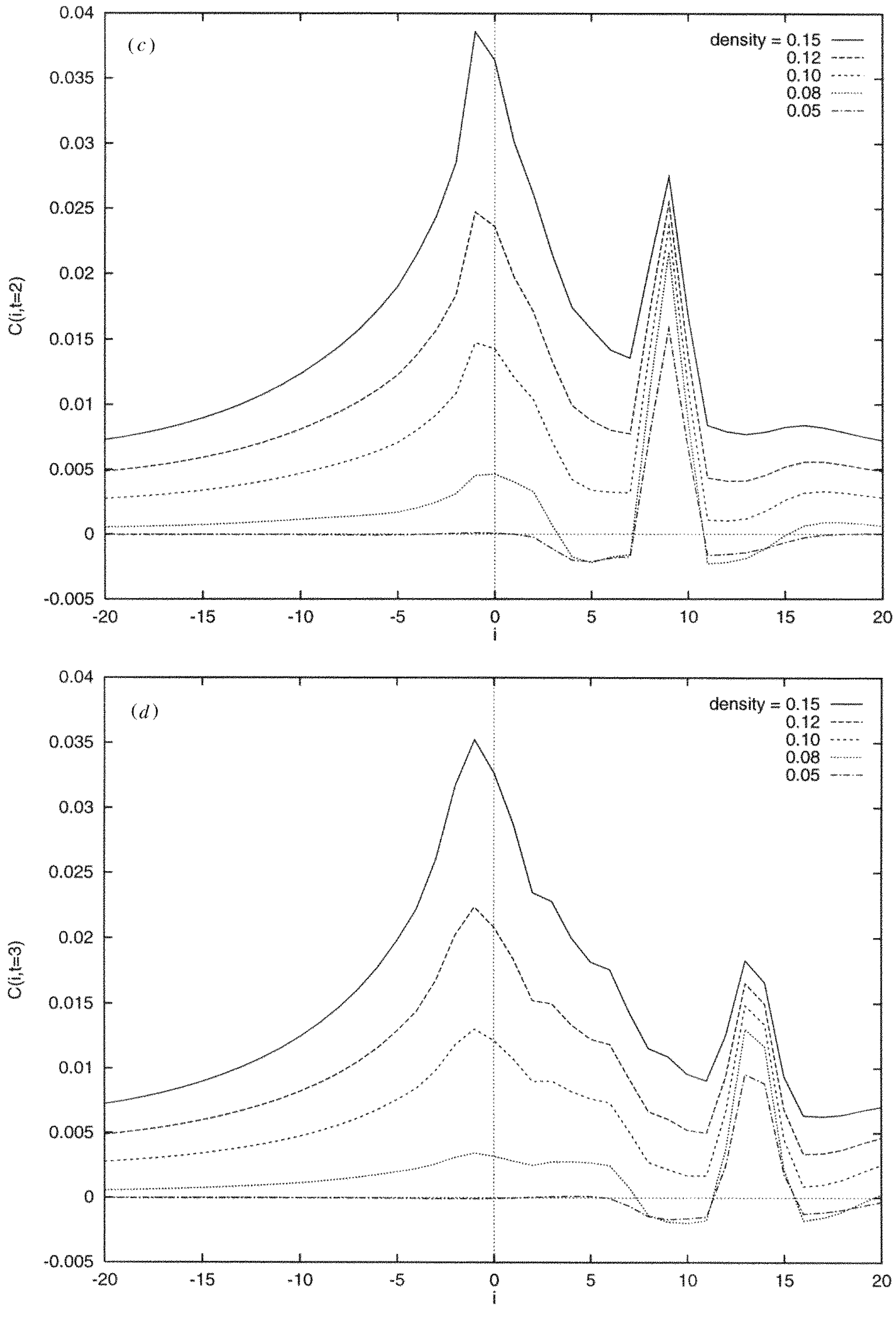


Figure 1. (Continued)

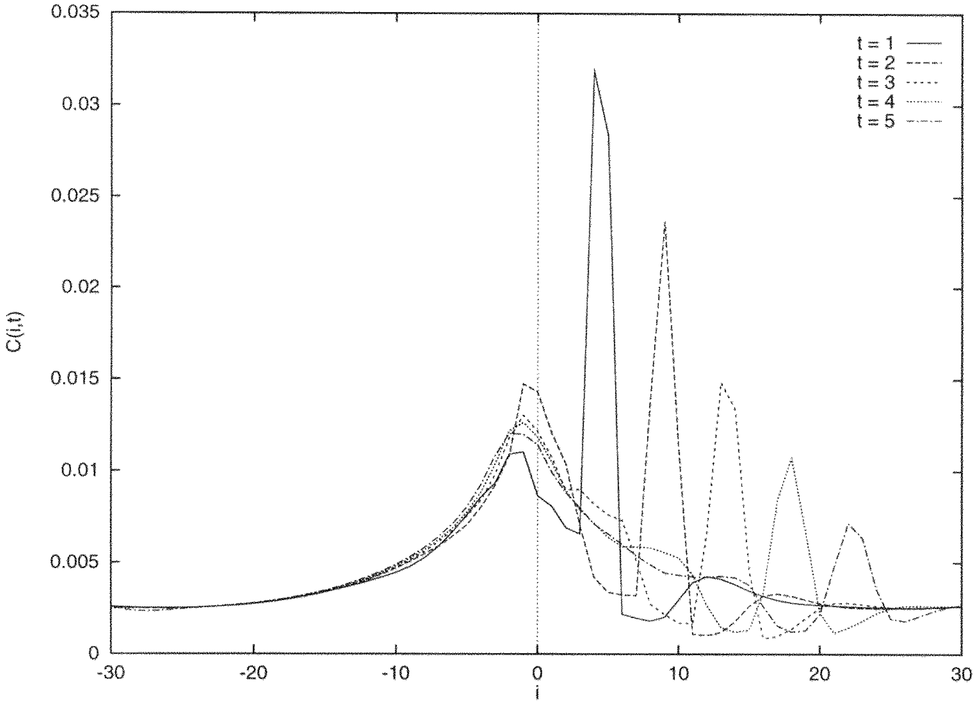


Figure 2. Correlation functions for different correlation times ($L = 4096$, $\rho = 0.1$). For increasing correlation times the autocorrelation peak moves to the right.

decreasing densities which is due to the fact that for very low densities the cars disperse over the system and their interaction with each other is very small.

In connection with the jamming–superjamming transition another interesting feature should be mentioned.

For $t = 0$ and high densities, there are minor maxima at $i = \pm 1$ (figure 4(c)); for $|i| \geqslant 2$ the correlation functions exponentially decay.

In order to understand this phenomenon we need a closer look at the deterministic case. As we can see from figure 4(c), the correlation functions show strong anticorrelations at $i = \pm 1$, which are maximally developed at the density $\rho = 0.5$. Also, with respect to the structure of the correlation functions, there is a striking relationship with the case $v_{\max} = 1$.

For $v_{\max} = 1$ we have a transition at $\rho = 0.5$. Corresponding to car–hole-symmetry, each car is surrounded by empty space for $\rho < 0.5$ whereas for $\rho > 0.5$ we have single holes surrounded by cars. Consequently, at $\rho = 0.5$ each cell is alternately occupied by a car or empty. At this density the correlation function $C_{v_{\max}=1, p=0}$ $(i, t = 0)$ shows a periodic structure: for $|i| = 2n$ we have maxima, for $|i| = 2n + 1$ minima (with $n \in \mathbb{Z}$). Both minima and maxima are maximally developed at $\rho = 0.5$.

As is well known for the case $v_{\max} = 5$, the cars arrange themselves at $\rho = \frac{1}{v_{\max}+1} = \frac{1}{6}$ in such a way that there are five empty cells between two cars each. This feature is inherited to higher densities (figure 4(c)). At $i = \pm 1$ there are strong anticorrelations. These are maximally developed between $\rho = 0.4$ and $\rho = 0.5$. For $|i| \geqslant 2$ $C(i, t = 0)_{p=0}$ decays although it still shows a structure similar to the case $v_{\max} = 1$.

Returning to $p = 0.5$ we see only traces of the anticorrelations at $i = \pm 1$. For $|i| \geqslant 2$ the correlation functions show no structure and just decay exponentially.

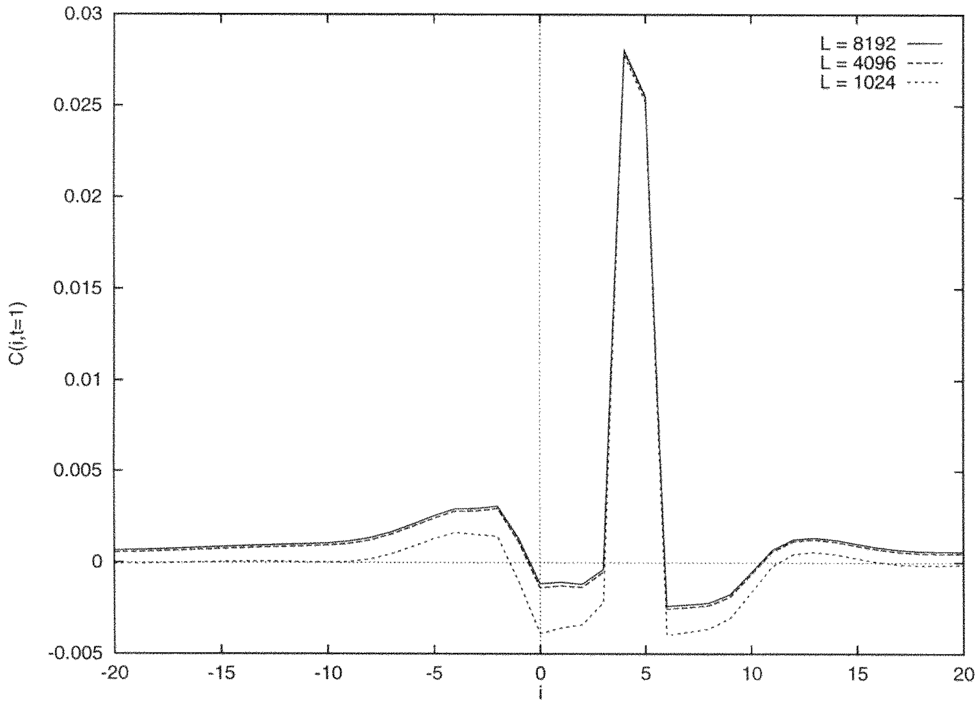


Figure 3. Correlation functions for different system sizes ($\rho = 0.08$). From $L = 4096$ on, no remarkable change takes place.

To simplify matters, we choose $i = \pm 2$ as a representative point in figure 6(b) (but what we observe there is also valid for $|i| > 2$). Interestingly, $C(i = \pm 2, t = 0)$ reaches its maximum at $\rho = 0.39$, which roughly coincides with the density where the propagating peak for $t > 0$ completely vanishes (figure 4(a)) and with the percolation density of Csányi and Kertész [7].

What figures 1(a)–(d) have in common, is that at very low densities and with increasing densities, anticorrelations develop around the propagating peak, that is to say, the effective repulsion due to the motion of the car is apparent. The reason for the anticorrelations is that a moving car has to have free space behind (where it comes from) and in front (in order to be able to move) and for low densities the cars self-organize in a way that they can move. Without random braking events this would lead to a perfect free flow below the density corresponding to the maximum in the fundamental diagram. However, due to the stochasticity of the NS model, at a density between $\rho = 0.07$ and 0.08 the above tendency changes and the anticorrelations start to *decrease* with increasing densities corresponding to the unavoidable disturbances caused by other cars. At even higher densities the anticorrelations vanish.

But before we come back to anticorrelations we need a closer look at the influence of the system size L on the correlation functions.

For very low and for very high densities the correlation functions for different system sizes L coincide. Only for densities near the transition from freely moving to jammed traffic are the curves different. In figure 3 we compare the correlation functions $C(i, t = 1)$ for the system sizes $L = 1024$, 4096, 8192 with each other ($\rho = 0.08$). It is obvious that for system sizes larger than $L = 4096$ no remarkable change takes place. Thus, correlation

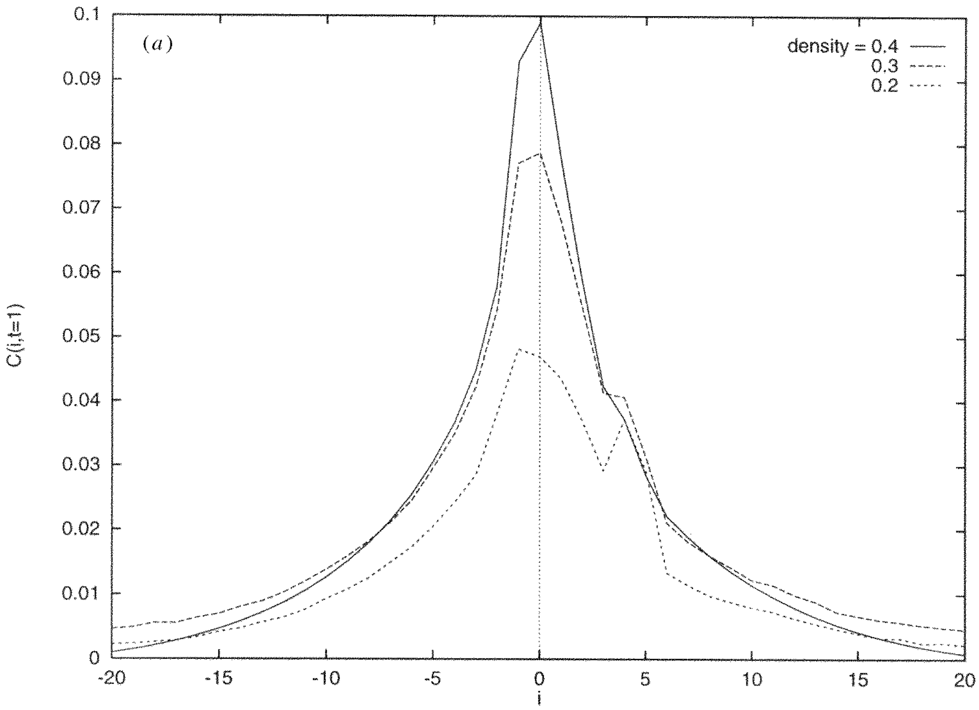


Figure 4. (*a*) Correlation functions for the car densities $\rho = 0.2$, 0.3, 0.4 and $t = 1$. Note that for $p = 0.3$ the cars are already in the superjam, although there are traces of the propagating peak. (*b*) Correlation functions for the car densities $\rho = 0.15$, 0.39, 0.41 and $t = 0$. We see that minor minima at $i = \pm 1$ appear for high densities. (*c*) Correlation functions for the deterministic case ($\rho = 0.4$, 0.5, 0.6, 0.7, 0.8 and $t = 0$). At $i = \pm 1$ anticorrelations can be obtained.

functions for $L = 4096$ can be considered as correlation functions in an infinite system in good approximation.

4. The effect of the jamming transition

One of the main advantages of the NS model is that in spite of its simplicity it captures the most important features of single-lane traffic flow including the transition from free flow to jamming. Recently it has been shown [18, 19] that the transition is not sharp but rather of the crossover type. With the above choice of the parameters p, $v_{\max}$ the transition can be located between $\rho = 0.071$ and 0.072. Interestingly, the densities where the anticorrelations are maximally developed and where the jam peak at $i = -1$ starts to form numerically coincide with this density. The origin of the jamming transition and that of these two signatures is the same: there is no more possibility for the cars to arrange themselves in a way where virtually no hindrance from each other occurs, they start to feel each other. As a consequence both signatures can be used as practical 'definitions' of the jamming transition.

In order to illustrate this we look at the minima of the anticorrelations as a function of the car density. Taking the derivatives of the resulting curves we get the diagrams shown in figure 5.

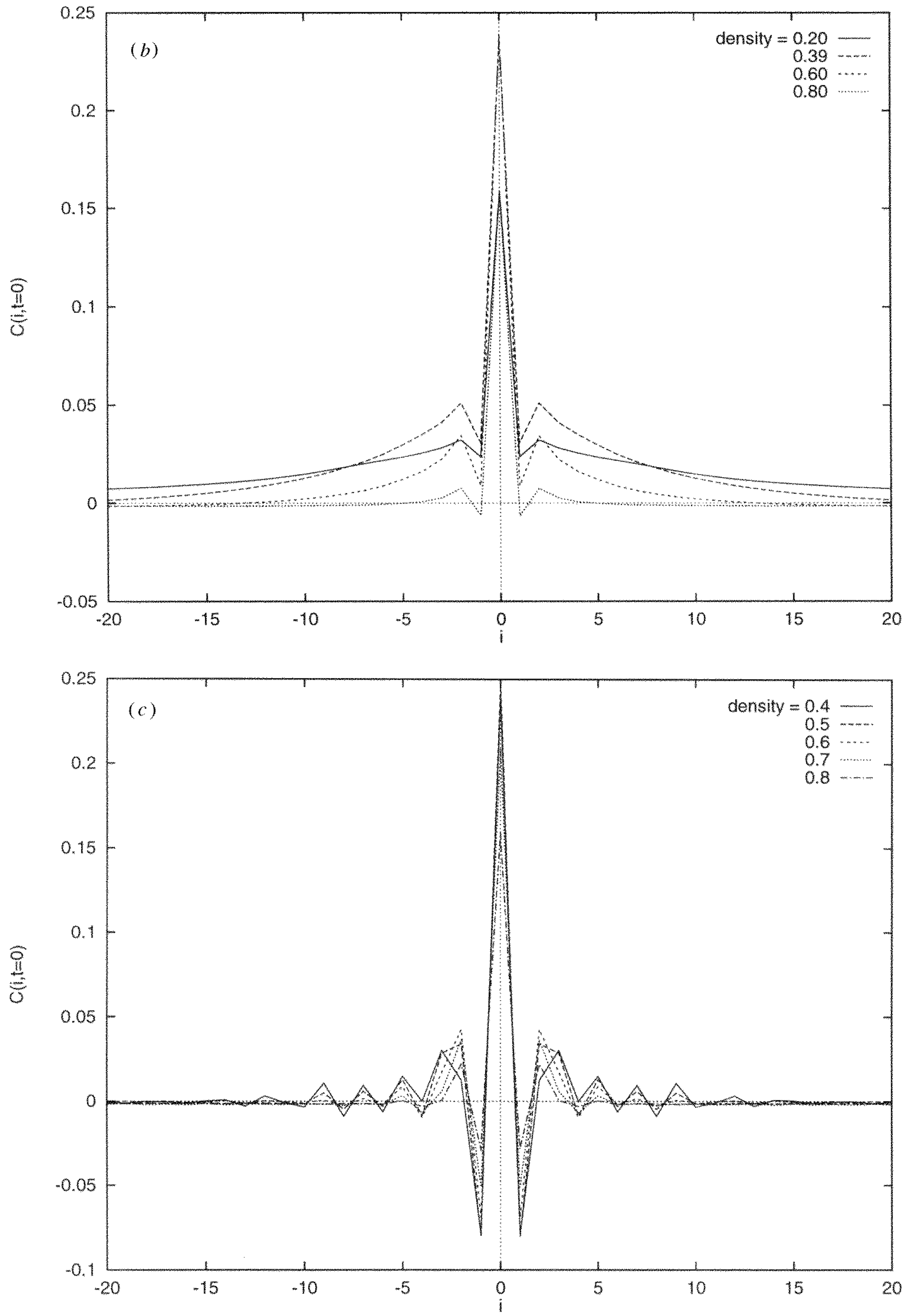


Figure 4. (Continued)

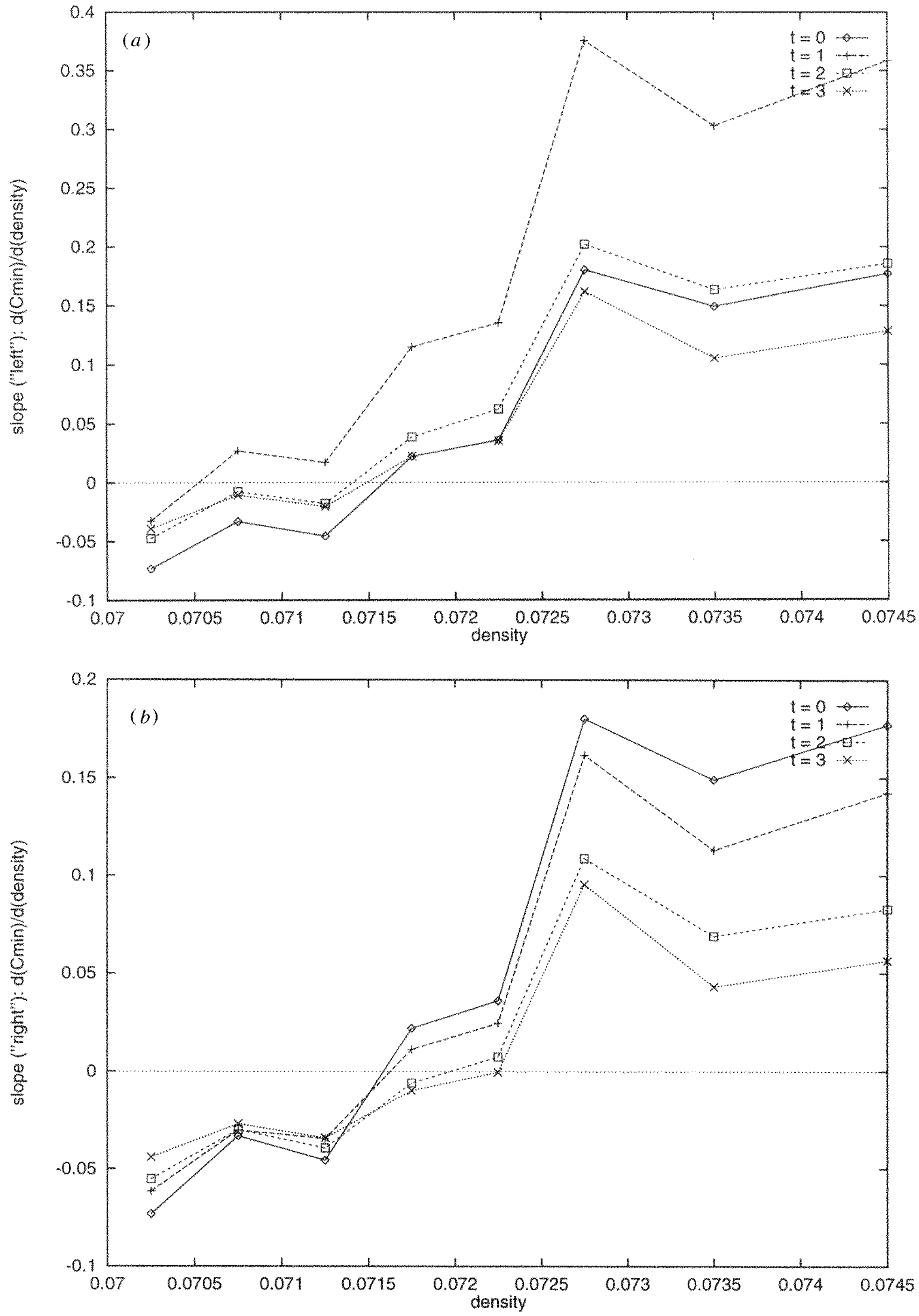


Figure 5. Slope of the minima of the anticorrelations as a function of the car density. In (*a*) ((*b*)) we consider the anticorrelations at $i = -1$ for $t = 0$ and $i = 5(t-1)$ for $t = 1, 2, 3$ (at $i = 5t + 1$ for $t = 0, 1, 2, 3$) describing the space to the front (back) car. According to our definition the transition from freely moving to jammed traffic takes place at the density where the curves in (*a*), (*b*) become zero. It is obvious that the density at which the transition happens is not clearly determined and therefore we do not have a phase transition.

5. The effect of the superjamming transition

In this paper the system is said to be superjammed if the correlation function for $t > 0$ is monotonously falling for all i with $t(v_{\max} - 1) \leqslant i \leqslant tv_{\max}$. It is true that traces of the propagating peak may occur in the superjammed region, too, but in this case we cannot call them a peak any more.

In order to get a precise definition for the transition from jamming to superjamming we denote the site where the propagating peak has its maximum at low densities with $i_{\max}$ and introduce

$$\mathrm{d}C = C(i_{\max}, t > 0) - C(i_{\max} - 1, t > 0).$$

According to this definition the jamming-superjamming transition takes place at $\mathrm{d}C = 0$. For $\mathrm{d}C > 0$ we have jamming, for $\mathrm{d}C < 0$ superjamming.

From figure 6(*a*) it is obvious that this transition is even more blurred than the transition from freely moving to jammed traffic. This is not only valid for $t = 1, 2, 3$ as shown in figure 6(*a*) but also for larger time differences.

6. Conclusions

By investigating the short-range correlation functions in the NS model we found that depending on the car density ρ the dynamics of the system can be divided into three regimes.

$0 < \rho \leqslant \rho_1$: free flow. The free flow is characterized by anticorrelations around a propagating peak, that is, in free flow moving cars are surrounded by empty space.

$\rho_1 < \rho \leqslant \rho_2$: jamming. The coexistence of free flow and jamming manifests itself in the double peak structure of the correlation function. The jamming causes a maximum at $i = -1$ according to the hindrance the back car feels in the jam.

$\rho_2 < \rho \leqslant 1$: superjamming. The propagating peak disappears as a consequence of the fact that in free flow moving cars do not exist any longer.

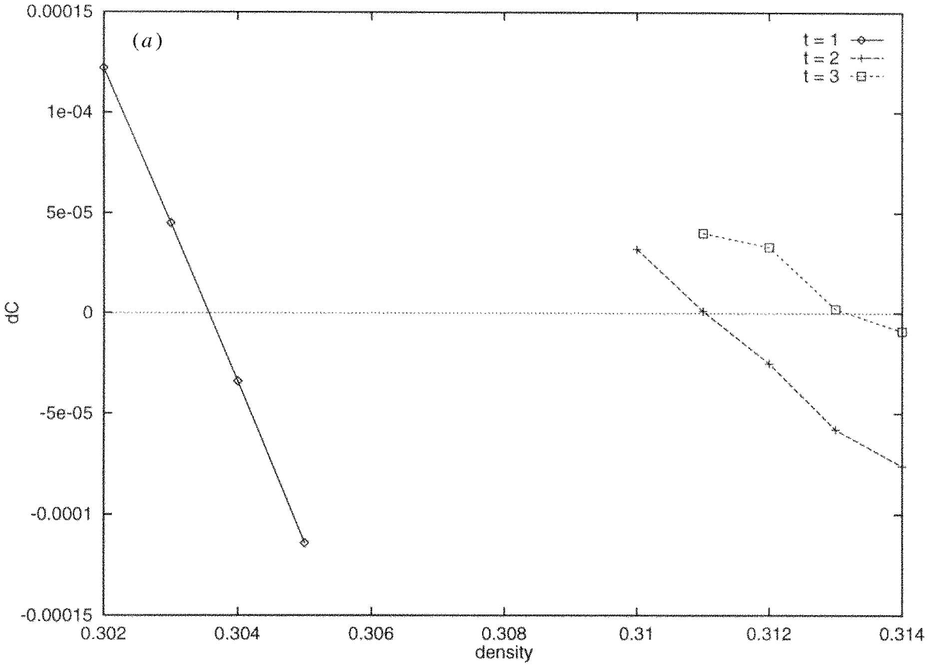
In order to define ρ_1 (ρ_2), where the transition from freely flowing (jammed) to jammed (superjammed) traffic takes place, we proceed as follows. We equate (as a practical definition) ρ_1 with the density where the anticorrelations are maximally developed and ρ_2 with the density where the correlation function $C(i, t)$ becomes monotonously falling for all i where for lower densities the propagating peak is observed.

On the basis of these definitions it turns out that both the transition from freely moving to jammed traffic and from jammed to superjammed traffic is not sharp.

The development of minor maxima for $t = 0$ in the superjamming region is a hint at the fact that single jamming waves become connected.

Furthermore, we showed that the short-range correlation functions are only slightly influenced by the system size L, so that an infinite system is well approximated by $L = 4096$.

Finally, we want to mention that the free flow-jamming and the jamming–superjamming transition differently manifest themselves in short-range correlation functions for $p = 0$. A



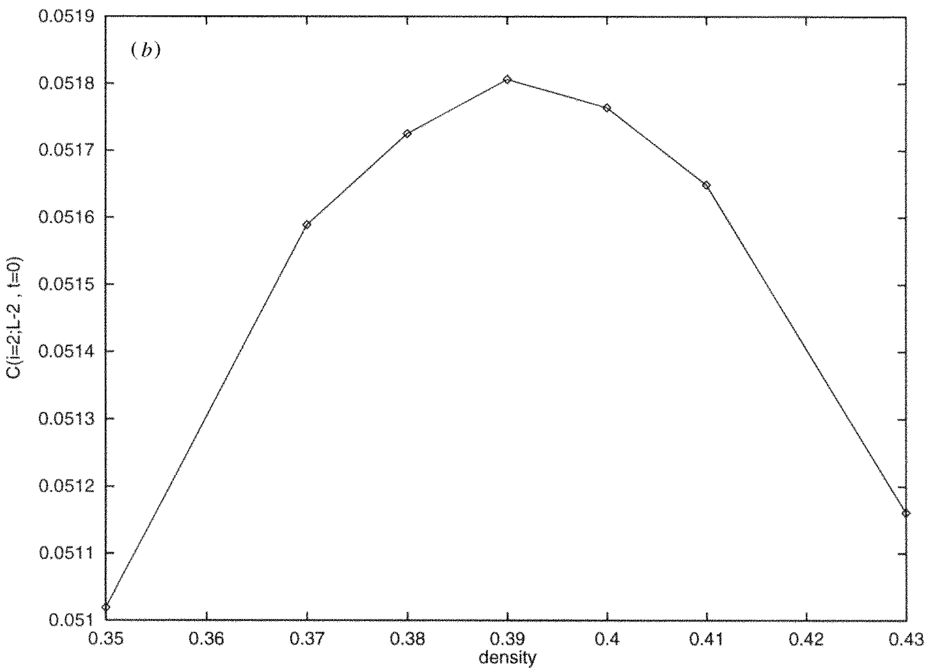


Figure 6. (a) $dC = C(i_{\max}, t > 0) - C(i_{\max} - 1, t > 0)$ as a function of the car density ($i_{\max}$: maximum of the propagating peak). Defining $dC = 0$ as the transition point we see that the transition from jammed to superjammed traffic is not sharp. (b) Correlation function for $t = 0$ and $i = \pm 2$ in dependence on the car density. We see that $C(i = \pm 2, t = 0)$ reaches its maximum at $\rho = 0.39$, where the correlation with the back and the front car are maximally developed.

detailed analysis of short-range correlation functions in the deterministic case, however, would go beyond the scope of this paper and will be published elsewhere.

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